

Prove the derivative of $\arccos x$ using implicit differentiation.

SCORE: ____ / 4 PTS

$$y = \cos^{-1} x$$

$$\cos y = x \quad \text{AND} \quad y \in [0, \pi] \rightarrow \sin y \geq 0$$

$$(-\sin y) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$= -\frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\text{SO, } \sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

Find $\frac{dy}{dx}$ if $(x^3y^2 - y^3 + 1)^2 = 2 - x^2$.

SCORE: ____ / 6 PTS

$$\underbrace{2(x^3y^2 - y^3 + 1)}_{\textcircled{1}} \left(\underbrace{3x^2y^2}_{\textcircled{\frac{1}{2}}} + \underbrace{x^3(2y)}_{\textcircled{1}} \frac{dy}{dx} - \underbrace{3y^2}_{\textcircled{1}} \frac{dy}{dx} \right) = \underbrace{-2x}_{\textcircled{\frac{1}{2}}}$$

$$3x^2y^2 + (2x^3y - 3y^2) \frac{dy}{dx} = -\frac{x}{x^3y^2 - y^3 + 1}$$

$$\frac{dy}{dx} = \left[\frac{-\frac{x}{x^3y^2 - y^3 + 1} - 3x^2y^2}{2x^3y - 3y^2} \right] \textcircled{1}$$

$$= \left[\frac{-x - 3x^2y^2(x^3y^2 - y^3 + 1)}{(2x^3y - 3y^2)(x^3y^2 - y^3 + 1)} \right] \textcircled{1}$$

$$\frac{d}{d\theta}(\sec \theta)^{\theta^2}$$

$$y = (\sec \theta)^{\theta^2}$$

$$\ln y = \theta^2 \ln \sec \theta \quad \textcircled{\frac{1}{2}}$$

$$\frac{1}{y} \frac{dy}{d\theta} = \underbrace{2\theta \ln \sec \theta}_{\textcircled{\frac{1}{2}}} + \underbrace{\theta^2 \frac{1}{\sec \theta} \sec \theta \tan \theta}_{\textcircled{2}}$$
$$\textcircled{\frac{1}{2}} = 2\theta \ln \sec \theta + \theta^2 \tan \theta \quad \textcircled{2}$$

$$\frac{dy}{d\theta} = y(2\theta \ln \sec \theta + \theta^2 \tan \theta)$$

$$= \underbrace{(\sec \theta)^{\theta^2}}_{\textcircled{1}} \underbrace{(2\theta \ln \sec \theta + \theta^2 \tan \theta)}_{\textcircled{\frac{1}{2}}}$$

SEE ALSO ALTERNATE SOL'N

(GRADE AGAINST ONE
VERSION OF SOLUTION
ONLY)

ALTERNATE SOLUTION

$$\frac{d}{d\theta} (\sec \theta)^{\theta^2}$$

$$= \frac{d}{d\theta} (e^{\ln \sec \theta})^{\theta^2}$$

$$= \frac{d}{d\theta} \underbrace{e^{\theta^2 \ln \sec \theta}} \textcircled{1}$$

$$= \underbrace{e^{\theta^2 \ln \sec \theta}} \textcircled{\frac{1}{2}} \left(\underbrace{2\theta \ln \sec \theta} \textcircled{\frac{1}{2}} + \theta^2 \underbrace{\frac{1}{\sec \theta} \sec \theta \tan \theta} \textcircled{2} \right)$$

$$= \underbrace{(\sec \theta)^{\theta^2}} \textcircled{\frac{1}{2}} \left(\underbrace{2\theta \ln \sec \theta + \theta^2 \tan \theta} \right) \textcircled{\frac{1}{2}}$$

$$\frac{d}{dt} \tan^{-1} \frac{t^2}{1-t}$$

$$= \left[\frac{1}{1 + \left(\frac{t^2}{1-t}\right)^2} \right] \cdot \left[\frac{2t(1-t) - t^2(-1)}{(1-t)^2} \right]$$

$$= \textcircled{2} \frac{2t - 2t^2 + t^2}{(1-t)^2 + t^4}$$

$$= \left[\frac{2t - t^2}{(1-t)^2 + t^4} \right] \textcircled{1}$$

$$\frac{d}{dy} \arcsin(e^{y^2} + \csc y)$$

$$= \frac{1}{\sqrt{1 - (e^{y^2} + \csc y)^2}} \cdot (e^{y^2} \cdot 2y - \csc y \cot y)$$

$$= \frac{\textcircled{2} \cdot 2y e^{y^2} - \csc y \cot y \textcircled{1}}{\sqrt{1 - (e^{y^2} + \csc y)^2} \textcircled{2}}$$

$$\frac{d}{dx} \ln \frac{\tan^4 x}{\sqrt[3]{x + \sin x}}$$

$$= \frac{d}{dx} (4 \ln \tan x - \frac{1}{3} \ln (x + \sin x))$$

$$= \frac{4 \sec^2 x}{\tan x} - \frac{1}{3} \frac{1}{x + \sin x} (1 + \cos x)$$

$$= \frac{4 \sec^2 x}{\tan x} - \frac{1 + \cos x}{3(x + \sin x)}$$